

PHYS103: GENERAL PHYSICS III 2 UNITS 2018/2019 SESSION
VISCOSITY, STOKES LAW, POISEUILLE'S LAW, REYNOLDS NUMBER,
TURBULENCE & LAMINAL FLOW

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VISCOSITY

Viscosity may be defined as the internal friction in a fluid. Both liquid and gases exhibit viscosity, although liquids are more viscous than gases. Fluid that flow readily, such a water or, have smaller viscosities than "thick" liquids such as honey or motor oil. Viscosity increases with increase in temperature for gases but decreases for liquids. The unit of viscosity is Ns/m^2 having dimension $\text{ML}^{-1}\text{T}^{-1}$. $1 \text{ Ns/m}^2 = 10 \text{ poise} = 1 \text{ dyne sec/cm}^2 = 1 \text{ poise}$.

The problem of viscous flow is similar to that of shear stress and strain in the solid. When a shear stress is applied to a solid the effect of the stress is to produce a certain displacement x of the solid, with fluid on the other hand, the shear strain increases without limit so long as the stress is applied and the shear stress is found to depend not on the shear strain but on the rate of change of shear strain. Hence we define the following:

$$\text{Shear strain } (\delta) = \frac{\text{displacement } (D)}{\text{Transverse dimension } (L)} \quad (1)$$

$$\text{Shear stress } (\tau) = \frac{\text{Force } (F)}{\text{Area } (A)} \quad (2)$$

$$\text{The rate of change of strain} = \frac{d}{dt} \left(\frac{\text{displacement } (D)}{\text{transverse dimension } (L)} \right) \quad (3)$$

$$\text{The rate of change of strain} = \frac{\text{velocity } (v)}{\text{transverse dimension } (L)} \quad (4)$$

$$\begin{aligned} \text{The coefficient of viscosity } (\eta) &= \frac{\text{Shear stress } (\tau)}{\text{The rate of change of strain}} \\ &= \frac{\text{Shear stress } (F/A)}{\text{velocity gradient } \left(\frac{v}{L}\right)} \end{aligned} \quad (5)$$

Hence, the force required for the motion is directly proportional to the velocity and is given by:

$$F = \eta A \frac{v}{L} \quad (6)$$

The coefficient of viscosity is obtained from equation (5) as

$$\eta = \frac{FL}{Av} \quad (7)$$

Equation (7) shows that the coefficient of viscosity depends on (i) Force F (ii) Transverse dimension L (iii) cross sectional area A (iv) velocity v (v) velocity gradient v/L . The coefficient of viscosity can be determined by (i) Stokes's law (ii) Poiseuille's law.

STOKE'S LAW AND TERMINAL VELOCITY

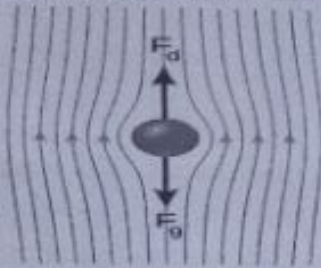
Stokes's law relate the viscous resisting force F_d , the weight W of the object and the Upthrust U it experiences as it falls through fluids when it has attained $F_d = W - U$. When a sphere move through an ideal but stationary fluid of zero viscosity, the streamline form a perfectly symmetrical pattern along the sphere. The pressure at any point on the upstream hemispherical

surface is exactly the same as that at the corresponding point on the downstream phase and the resultant force on the sphere is zero. A complete analysis by Stokes's law shows that viscous resisting force

$$F_d = 6\pi\eta rv \quad (8)$$

where

η = coefficient of viscosity, v = velocity with which the solid sphere move through a fluid (m/s), r = radius of the solid sphere. Consider a sphere falling through a viscous fluid. As the sphere falls so its velocity increases until it reaches a velocity known as the terminal velocity V_T .



At this velocity, the viscous drive force plus the upthrust U on the sphere equals the weight of the sphere.

Let ρ = density of the solid and ρ' = density of the liquid
hence

$$F_d = W - U$$

$$U = \frac{4}{3}\pi r^3 \rho' \times g$$

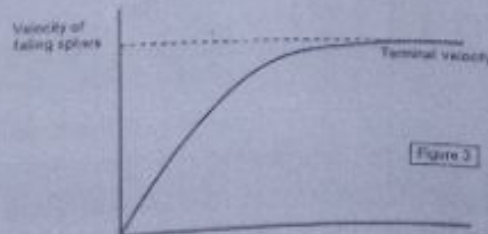
$$W = \frac{4}{3}\pi r^3 \rho \times g$$

$$6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g$$

$$6\pi\eta rv = \frac{4}{3}\pi r^3 g (\rho - \rho')$$

$$V_T = \frac{2gr^2}{9\eta} (\rho - \rho') \quad (9)$$

Equation (9) is the terminal velocity of a sphere falling through a viscous fluid. Where g is the acceleration due to gravity. A terminal velocity is the velocity attained by a falling bodies in a fluid when the net force of the viscous resisting force F_d , its weight W and upthrust U is zero. $W - U - F_d = 0$.



At higher velocity Stokes's law does not valid because the fluid becomes turbulent. Stokes' law makes the following assumptions for the behavior of a particle in a fluid:

- i. Laminar Flow
- ii. Spherical particles
- iii. Homogeneous (uniform in composition) material
- iv. Smooth surfaces
- v. Particles do not interfere with each other.

Question

Determine the viscous resisting force due to a fluid (glycerine) if it has an area of $2 \times 10^{-3} \text{ m}^2$ in a region of velocity gradient 0.5 s^{-1} and whose coefficient of viscosity is 0.35 Ns/m^2 (3.5 poise at 30°C).

Solution:

$$F = \eta A \frac{v}{L} = 0.35 \times 2 \times 10^{-3} \times 0.5 = 3.5 \times 10^{-4} \text{ N}$$

POISEUILLE'S EQUATION OR LAW

Poiseuille's Equation or Law states that the volume flow rate (Q) of an incompressible fluid of viscosity, (η) through a pipe of internal radius (R) and length (L) is proportional to the pressure gradient.

Mathematically:

$$Q = \frac{\pi R^4 \Delta P}{8\eta L} \quad (10)$$

This equation shows that the volume flow rate is very sensitive to changes in the radius of the pipe. Poiseuille's Equation holds for an incompressible fluid (Fluid of constant density) or Newtonian fluid flowing in a pipe. Non-Newtonian fluid do not obey Poiseuille's law because their viscosities are velocity dependent. The assumption of streamlined (laminar) flow is built in Poiseuille's law. In a capillary flow, the pressure difference ΔP across the ends of the tube is given as:

$$\Delta P = h\rho g \quad (11)$$

where h is the height of the fluid, ρ is the density of the fluid and g is the acceleration due to gravity. Using equation (11) in equation (10), the expression for the coefficient of viscosity may be written as:

$$\eta = \frac{\pi R^4 h\rho g}{8Q L} \quad (12)$$

By applying Ohm's law in electric circuit of potential V and current I , the resistance (R_p) to the flow of fluid of coefficient of viscosity (η) in a pipe of length (L), internal radius (R), volume flow rate Q and pressure difference ΔP is given by

$$R_p = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4} \quad (13)$$

The energy is dissipated when a fluid flows through a pipe. This is manifested by the drop in pressure along the pipe.

Effects of Poiseuille's Equation or Law

1. The flow of gas in respiratory system is poiseuillean. When resistance (any constriction of the tube) to the flow is increased, the breathing becomes more difficult (Asthma). The pressure gap also increase and cause strain on the heart muscles, so heart has to work harder to compensate. And at times of stress, when an increased flow rate is required, there can be a breakdown.

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2. Water will rise quicker in large grains soils ($Q \propto R^4$) but it will rise to a greater height by capillary attraction on fine grain soil ($h \propto 1/R$).
 3. Poiseuille's equation is considered in the design of plumbing system and hypodermic needles. The needle size determine the flow rate than the thumb pressure. Doubling the needle diameter has the same effect as increasing the thumb pressure sixteen fold.
 4. Irrigation pipes are constructed closer to the river so as to reduce the resistance. The resistance of the pipe increases with length and thus one need a very big pump.

THE REYNOLDS NUMBER (Re)

In fluid mechanics, the **Reynolds number (Re)** is the ratio of **momentum forces** (inertial force) to **viscous forces** (shearing force). That is, how fast the fluid is moving relative to how viscous the fluid is, irrespective of the scale of the fluid system. It is a dimensionless quantity. The concept was introduced by George Gabriel Stokes in 1851 but the Reynolds number is named after Osborne Reynolds (1842–1912). The Reynolds number is an important parameter that characterize whether flow conditions lead to laminar or turbulent flow:

- *Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion;*
- *Turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.*

Mathematically,

$$\text{Reynolds number (Re)} = \frac{\text{inertial forces (F}_i\text{)}}{\text{viscous forces (F}_v\text{)}} = \frac{\rho v D}{\mu} = \frac{v D}{\tau} = \frac{Q D}{\tau A} \quad (14)$$

where all the parameters are defined as follows:

D =hydraulic diameter of the pipe, it is linear dimension or travelled length of the fluid in metre (m)

Q =volumetric flow rate (m^3/s), A =cross-sectional area (m^2) of the pipe, v =mean velocity of the fluid (m/s)

μ =dynamic viscosity of the fluid ($\text{Pa}\cdot\text{s} = \text{N}\cdot\text{s}/\text{m}^2 = \text{kg}/\text{m}\cdot\text{s}$), τ =kinematic viscosity of the fluid ($\tau = \mu/\rho$) in (m^2/s)

ρ =density of the fluid (kg/m^3)

Whenever the **Reynolds number** is less than about 2,000, flow in a pipe is generally **laminar**, whereas, at values greater than 2,000, flow is usually turbulent. As the Reynolds number increases, such as by increasing the flow rate of the fluid, the flow will experience transition from laminar to turbulent flow at a specific range of Reynolds numbers, the laminar-turbulent transition range depending on small disturbance levels in the fluid or imperfections in the flow system. If the Reynolds number is very small, much less than 1, then the fluid will exhibit Stokes or creeping flow.



The flow of water from a faucet (water valve) is
 (a) Laminar at low speeds but (b) turbulent at sufficiently high speeds

Turbulence

In fluid dynamics, turbulence or turbulent flow is a flow pattern that is very chaotic (not steady), irregular in both space and time (flow pattern changes continuously). It occurs when boundary surfaces cause abrupt changes in velocity and characterized by local, short-lived rotation currents known as vortices. Turbulent flow is a less orderly flow regime that is characterized by eddies or small packets of fluid particles. It is also noticeable when there is disturbances in air flows. The Reynolds number for turbulent flow is ranges between 2000 to 2500. This characterized by rapid variation of pressure and high velocity in space and time.

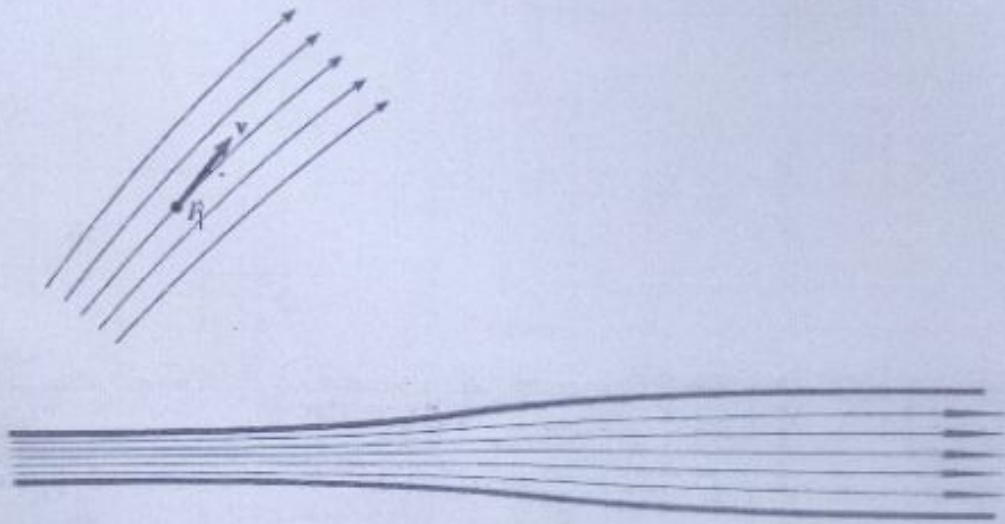
Examples of Turbulence

1. Clear-air turbulence experienced during airplane flight, as well as poor astronomical seeing (the blurring of images seen through the atmosphere.)
2. Smoke rising from a cigarette: For the first few centimeters, the flow is certainly laminar. The smoke becomes turbulent as its Reynolds number increases, as its flow velocity and characteristic length are both increasing.
3. The flow conditions in many industrial equipment (such as pipes, ducts, precipitators, gas scrubbers, dynamic scraped surface heat exchangers, etc.) and machines (for instance, internal combustion engines and gas turbines).
4. The external flow over all kind of vehicles such as cars, airplanes, ships and submarines.
5. The motions of matter in stellar atmospheres.
6. Bridge supports (piers) in water. In the late summer and fall, when river flow is slow, water flows smoothly around the support legs. In the spring, when the flow is faster, a higher Reynolds Number is associated with the flow. The flow may start off laminar but is quickly separated from the leg and becomes turbulent.
7. In the medical field of cardiology, a stethoscope is used to detect heart sounds and bruits, which are due to turbulent blood flow. In normal individuals, heart sounds are a product of turbulent flow as heart valves close. Turbulence makes noise

Laminar flow of fluid

This is a flow pattern of fluid in which the fluid travels smoothly in regular paths. It is a steady flow. The velocity, pressure, and other flow properties at each point in the fluid remain

constant, in contrast to turbulent flow, in which the fluid undergoes irregular fluctuations and mixing. A particle in laminar flow follows a streamline, and at each point along its path the particles velocity is tangential to the streamline



Laminar flow

When a fluid is flowing through a closed channel such as a pipe or between two flat plates. The two types of flow may occur depending on the **velocity** and **viscosity** of the fluid: **laminar flow** or turbulent flow. **Laminar flow** tends to occur at **lower velocities**, below a threshold at which it becomes turbulent. Laminar flow over a horizontal surface may be thought of as consisting of thin layers, all parallel to each other, that slide over each other. It is common only where the flow channel is relatively small, the fluid is moving slowly, and its viscosity is relatively high. **Examples** include the flow of oil through a thin tube and normal blood flow in the human aorta.